Help message for MATLAB function *refval2.m*

```
>> help refval2
  Function refval2 determines the geometric and physical constants of the
   equipotential rotational reference ellipsoid of the World Geodetic
   System 1984 (WGS84) defined by:
                    a - semi-major axis,
                  1/f - reciprocal of flattening
                omega - angular velocity of the Earth
                   GM - geocentric gravitational constant of the Earth,

  For WGS84 the defining parameters are
                    a = 6378137 m
                  1/f = 298.257223563
                omega = 7.292115e-05 rad/s
                   GM = 3.986004418e+14 m^3/s^2
```

Output from MATLAB function *refval2.m*

```
>> refval2


WORLD GEODESTIC SYSTEM 1984
===========================
Defining Constants (exact)

    a = 6378137 m                  semi-major axis
  1/f = 298.257223563              reciprocal of flattening
omega = 7.292115e-05 rad/s         angular velocity
   GM = 3.986004418e+14 m^3/s^2    geocentric gravitational constant

Derived Geometrical Constants of Normal Ellipsoid

    b =  6356752.314245 m       semi-minor axis
    E =   521854.008423 m       linear eccentricity
    c =  6399593.625758 m       polar radius of curvature
   e2 = 6.694379990141e-03      eccentricity squared
  ep2 = 6.739496742276e-03      2nd eccentricity squared
    f = 3.352810664747e-03      flattening
    n = 1.679220386384e-03      3rd flattening
    Q = 10001965.729313 m       quadrant distance
   R1 =  6371008.771415 m       mean radius R1 = (2a+b)/3
   R2 =  6371007.180918 m       radius of sphere of same surface area
   R3 =  6371000.790009 m       radius of sphere of same volume

Derived Physical Constants of Normal Ellipsoid

   U0 = 62636851.714569 m^2/s^2 normal gravity potential
   C2 = +4.841667749599e-04     2nd degree zonal harmonic (fully normalized)
   J2 = +1.082629821257e-03     2nd degree zonal harmonic (conventional)
    m =  3.449786506841e-03
  g_e =  9.780325335903 m/s^2   normal gravity at equator
  g_p =  9.832184937865 m/s^2   normal gravity at pole
   f* =  0.005302441399         gravity flattening
    k =  0.001931852653         constant in Pizzetti's equation

>>
```

MATLAB function *refval2.m*

```
function refval2
% Function refval2 determines the geometric and physical constants of the
%  equipotential rotational reference ellipsoid of the World Geodetic
%  System 1984 (WGS84) defined by:
%                    a - semi-major axis,
%                  1/f - reciprocal of flattening
%                omega - angular velocity of the Earth
%                   GM - geocentric gravitational constant of the Earth,
%
% For WGS84 the defining parameters are
%                    a = 6378137 m
%                  1/f = 298.257223563
%                omega = 7.292115e-05 rad/s
%                   GM = 3.986004418e+14 m^3/s^2

%==========================================================================
% Function: refval2
%
% Usage:   refval2;
%
% Author:
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%  RMIT University,
%  GPO Box 2476V, MELBOURNE VIC 3001
%  AUSTRALIA
%  email: rod.deakin@rmit.edu.au
%
% Date:
%  Version 1   21 May 2014
%
% Functions Required:
%  None
%
% Remarks:
%  Function refval2() determines the geometric and physical constants of an
%  equipotential rotational reference ellipsoid defined by:
%               a     - semi-major axis,
%             1/f     - reciprocal of flattening
%               omega - angular velocity of the Earth.
%               GM    - geocentric gravitational constant of the Earth,
%
%  Given the four defining parameters above, geometric and physical
%  constants can be derived by methods and formulae set out in Ref[1].
%  Theory is given in Ref[2] and Ref[4].
%
%  The following geometric constants of the ellipsoid are evaluated
%    b       - semi-minor axis of ellipsoid (metres)
%    c       - polar radius of curvature (metres)
%    C2      - fully normalized, second degree zonal harmonic:
%              C2 = J2/sqrt(5)
%    e2      - eccentricity squared: e = E/a
%    ep2     - second eccentricity squared: ep = E/b
%    E       - linear eccentricity: E = sqrt(a^2 - b^2) = a*e
%    f       - flattening of ellipsoid
%    J2      - conventional second degree zonal harmonic:
%              J2 = sqrt(5)*C2  J2 is also known as the dynamical form
%              factor
%    n       - third flattening of ellipsoid: n = f/(2-f)
%    Q       - quadrant distance of ellipsoid (metres)
%    R1      - radius of sphere having mean radius: R1 = (2*a + b)/3
%    R2      - radius of sphere having same surface area of ellipsoid
%    R3      - radius of sphere having same volume of ellipsoid
%
%  The following physical constants of the ellipsoid are evaluated
%    gamma_e - normal gravity at equator (m/s^2)
%    gamma_p - normal gravity at pole (m/s^2)
%    J4,J6,J8,... - coefficients of Legendre polynomials in the spherical
%                   harmonic expansion of the gravitational potential V.
%    k       - constant in Pizetti's equation for normal gravity.
%    m       - a derived physical constant: m = omega^2*a^2*b/GM
```

```
%    q0      - q(subscript zero), physical constant used in computation of
%              gamma_e, gamma_p
%    qp0     - q-primed(subscript zero), physical constant used in
%              computation of gamma_e, gamma_p
%    U0      - normal gravity potential at ellipsoid (m^2/s^2)
%
% Variables:
%  a       - semi-major axis of ellipsoid (metres)
%  b       - semi-minor axis of ellipsoid (metres)
%  c       - polar radius of curvature (metres)
%  c0      - coefficient in computation of quadrant distance Q
%  C2      - fully normalized second degree zonal harmonic
%  e       - (first) eccentricity of ellipsoid: e = E/a
%  e2      - (first) eccentricity squared
%  e21     - approximate value of e2
%  ep      - second eccentricity: ep = E/b
%  ep2     - second eccentricity squared
%  E       - linear eccentricity: E = sqrt(a^2 - b^2) = a*e
%  f       - flattening of ellipsoid: f = (a-b)/a = 1-sqrt(1-e^2)
%  f1      - gravity flattening
%  flat    - denominator of flattening: f = 1/flat
%  gamma_e - normal gravity at equator (m/s^2)
%  gamma_p - normal gravity at pole (m/s^2)
%  GM      - geocentric gravitational constant (m^3/s^2)
%  i,j     - integer counters
%  J2      - dynamical form factor (conventional 2nd degree zonal harmonic)
%  k       - constant in Pizetti's equation for normal gravity.
%  m       - a derived physical constant: m = omega^2*a^2*b/GM = m1/a*b;
%  m1      - a derived physical constant: m1 = omega^2*a^3/GM = m/b*a;
%  n       - third flattening of ellipsoid: n = f/(2-f)
%  n2,n4,... even powers of n
%  N       - size of vector Jvec
%  omega   - angular velocity of earth (radians/sec)
%  q0      - q(subscript zero), physical constant used in computation of
%            gamma_e, gamma_p
%  qp0     - q-primed(subscript zero), physical constant used in
%            computation of gamma_e, gamma_p
%  Q       - quadrant distance of ellipsoid (metres)
%  R1      - radius of sphere having mean radius: R1 = (2*a + b)/3
%  R2      - radius of sphere having same surface area as ellipsoid
%  R3      - radius of sphere having same volume as ellipsoid
%  sgn     - sgn = 1 or sgn = -1
%  U0      - normal gravity potential at ellipsoid (m^2/s^2)
%  x       - local variable
%
% References:
%  1. Moritz, H., 1980, 'Geodetic Reference System 1980',  Bulletin
%        Geodesique Vol.54(3), 1980, pp.395-405.
%  2. Heiskanen, W.A. and Moritz, H. (1967).  Physical Geodesy, W.H.Freeman
%        and Co., London, 364 pages.
%  3. World Geodetic System 1984, NIMA TR8350.2 Amendment 1, 03-Jan-2000,
%        National Imagery and Mapping Agency (NIMA), Technical Report
%        TR 8350.2
%  4. Deakin, R.E., 1997. 'The Normal Gravity Field', Private Notes,
%        Department of Geospatial Science, RMIT University, 38 pages.
%  5. Deakin, R.E. & Hunter, M.N., 2012. 'A Fresh Look at the UTM
%        Projection: Karney-Krueger equations', Presented at the Surveying
%        and Spatial Sciences Institute (SSSI) Land Surveying Commission
%        National Conference,  Melbourne, 18-21 April, 2012.
%  6. Deakin, R.E. & Hunter, M.N., 2013. 'Geometric Geodesy Part A', School
%        of Mathematical & Geospatial Sciences, RMIT University, 3rd
%        printing, January 2013.
%============================================================================

%-------------------------------------------------------------------------
% set defining constants of rotational equipotential ellipsoid for WGS84
%-------------------------------------------------------------------------
% see Ref[3], Table 3.1, p.3-5
a = 6378137.0;
flat = 298.257223563;
omega = 7.292115e-05;
GM = 3.986004418e+14;
```

```
%-------------------------------------------------------
% compute GEOMETRIC constants of equipotential ellipsoid
%-------------------------------------------------------
f = 1/flat;                    % flattening
e2 = f*(2-f);                  % eccentricity squared
e    = sqrt(e2);               % eccentricity of ellipsoid
ep2  = e2/(1-e2);              % 2nd eccentricity squared
ep   = sqrt(ep2);              % 2nd eccentricity
b    = a*(1-f);                % semi-minor axis of ellipsoid
E    = a*e;                    % linear eccentricity
c    = a*a/b;                  % polar radius of curvature
n    = f/(2-f);                % 3rd flattening

% computation of quadrant length of ellipsoid.  See Ref[5], eq (39).
n2 = n*n;      % even powers of n
n4 = n2*n2;
n6 = n4*n2;
n8 = n6*n2;
c0 = 1 + 1/4*n2 + 1/64*n4 + 1/256*n6 + 25/16384*n8;
Q = a/(1+n)*c0*(pi/2);       % quadrant distance

% computation of radii of equivalent spheres.  See Ref[6], pp. 86-87.
R1 = (2*a+b)/3;
R2 = sqrt(a^2/2*(1+(1-e2)/(2*e)*log((1+e)/(1-e))));
R3 = (a^2*b)^(1/3);

%------------------------------------------------------
% compute PHYSICAL constants of equipotential ellipsoid
%------------------------------------------------------

% compute normal potential of the reference ellipsoid U0
% See Ref[1] pp.399-400, Ref[2] p.67, eq.(2-61), Ref[4] eq.(38b)
U0 = GM/E*atan(ep) + omega^2*a^2/3;

% compute normal gravity at equator and poles
% formula for q0 given in Ref[2] eq.(2-58), p.66 and Ref[4] eq.(31), p.12
% see also Ref[1] p.398
q0 = ((1+(3/ep2))*atan(ep)-(3/ep))/2;
% formula for qp0 given in Ref[2] eq.(2-67), p.68 and Ref[4] eq.(52), p.19
% see also Ref[1] p.400; Ref[4] eq.(52)
qp0 = 3*(1+(1/ep2))*(1-(1/ep*atan(ep))) - 1;
% compute normal gravity at the equator and the pole
% formulae for gamma_e and gamma_p given in Ref[2] eqs (2-73) and (2-74),
% p.69; Ref[1] p.400 and Ref[4], eqs (62a),(62b).
m       = omega^2*a^2*b/GM;
gamma_e = GM/a/b*(1-m-(m/6*ep*qp0/q0));
gamma_p = GM/a/a*(1+(m/3*ep*qp0/q0));
% compute gravity flattening
f1 = gamma_p/gamma_e - 1;
% compute constant k in Pizetti's equation for normal gravity
% formula for k in Ref[1], p.401 and Ref[4] eq.(68), p.26.
k = b*gamma_p/(a*gamma_e) - 1;

%---------------------------------
% compute dynamical form factor J2
%---------------------------------
% J2 is also the conventional 2nd degree zonal harmonic
% see Ref[4], eq.(87), p.35
J2 = e2/3*(1-2/15*m*ep/q0);

%--------------------------------------------------------
% compute fully normalized second degree zonal harmonic C2
%--------------------------------------------------------
% see Ref[3], p.5-3 where the relationships between conventional and
% full-normalized gravitational potential coefficients is given.  For the
% case n=2, m=0, k=1 then FULLY-NORMALIZED = CONVENTIONAL/sqrt(5)
C2 = J2/sqrt(5);

% print headings and computed data
fprintf('\n\nWORLD GEODESTIC SYSTEM 1984');
fprintf('\n===========================');
fprintf('\nDefining Constants (exact) \n');
fprintf('\n    a = %7.0f m                  semi-major axis',a);
fprintf('\n  1/f = %12.9f              reciprocal of flattening',flat);
fprintf('\nomega = %9.6e rad/s         angular velocity',omega);
fprintf('\n   GM = %12.9e m^3/s^2    geocentric gravitational constant',GM);
```

```
fprintf('\n\nDerived Geometrical Constants of Normal Ellipsoid \n');
fprintf('\n    b = %15.6f m       semi-minor axis',b);
fprintf('\n    E = %15.6f m       linear eccentricity',E);
fprintf('\n    c = %15.6f m       polar radius of curvature',c);
fprintf('\n   e2 = %15.12e      eccentricity squared',e2);
fprintf('\n  ep2 = %15.12e      2nd eccentricity squared',ep2);
fprintf('\n    f = %15.12e      flattening',f);
fprintf('\n    n = %15.12e      3rd flattening',n);
fprintf('\n    Q = %15.6f m       quadrant distance',Q);
fprintf('\n   R1 = %15.6f m       mean radius R1 = (2a+b)/3',R1);
fprintf('\n   R2 = %15.6f m       radius of sphere of same surface area',R2);
fprintf('\n   R3 = %15.6f m       radius of sphere of same volume',R3);

fprintf('\n\nDerived Physical Constants of Normal Ellipsoid \n');
fprintf('\n   U0 = %15.6f m^2/s^2 normal gravity potential',U0);
fprintf('\n   C2 = %+15.12e     2nd degree zonal harmonic (fully normalized)',C2);
fprintf('\n   J2 = %+15.12e     2nd degree zonal harmonic (conventional)',J2);
fprintf('\n    m = % 15.12e',m);
fprintf('\n  g_e = %15.12f m/s^2   normal gravity at equator',gamma_e);
fprintf('\n  g_p = %15.12f m/s^2   normal gravity at pole',gamma_p);
fprintf('\n   f* = %15.12f         gravity flattening',f1);
fprintf('\n    k = %15.12f         constant in Pizzetti''s equation',k);


fprintf('\n\n');
```